Optimization of Thermal Efficiency of Buildings

By

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Abstract

In the building sector, the choice of the materials utilized in building the wall is dependent on many factors. Some of the factors that affect the choice of the materials are the bearing ability, water resistance, available thicknesses, cost, heat transfer, availability in the market and other factors. Designers always set one of these factors as their primary objective, while setting the others as subsidiary objectives of less importance which may be neglected in some cases. The goal of this research is to develop an approach for selecting the materials needed for building laminate composite walls that can achieve two objectives at the same time, which are minimum cost and minimum heat transfer through the wall. While considering the bearing conditions of the materials for the wall to be able to withstand the loads acting on it, and considering the required thickness constraint of the wall. The approach is divided to two steps, starting with the optimization part that gives the optimum set of materials for each number of layers separately, and then this optimum set of materials is used to calculate the accompanied transient heat transfer through the wall based on the location and orientation of the wall. The cost and actual heat transfer for each number of layers is finally presented to the designer to select the most suitable alternative. This approach would help construction designers for building thermally and economically efficient walls for residential or non-residential buildings, and can also be used in building walls of furnace or in building chambers where thermal efficiency is of major importance.
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Nomenclature

A  Surface area being studied/ ft\(^2\)
B\(_C\)  Value below the optimum cost
B\(_H\)  Value below the optimum heat blockage
bn, cn, dn  Conduction transfer function coefficients
CM\(_j\)  A factor used in the thermal model for each wall “j” to represent \(\frac{\rho_j A_j L_j C_{pj}}{dt}\)
C\(_P\)  Thermal Capacity
C\(_j\)  The corresponding cost of arc j in the network.
d\(_t\)  Time interval used in transient heat transfer calculations
E\(_{in}\)  Total amount of energy absorbed by the object
E\(_{out}\)  Total amount of energy released by the object
F\(_b\)  Compressive strength of the material in water saturation state
F\(_g\)  Compressive strength of the material in dry state
h  Combined coefficient of convection and radiation at a surface
h\(_{conv}\)  Convection heat transfer coefficient at a surface
h\(_i\)  Convection heat transfer coefficient at the surface of the inner wall
h\(_o\)  Convection heat transfer coefficient at the surface of the outer wall
h\(_{rad}\)  Radiation heat transfer coefficient at a surface
i  Index of the nodes in the network
I\(_G\)  Global solar radiation (irradiance) = 1.15*(SHGF)”Solar Heat Gain Factor”
j  Index of the arcs in the network
K  Conduction heat transfer coefficient
K\(_r\)  Softening coefficient of the material
L  Half the thickness of the wall/layer
M  Number of nodes in the network
N  Number of arcs in the network
O\(_C\)  Value over the optimum cost
O\(_H\)  Value over the optimum heat blockage
R\(_{cond}\)  Conduction thermal resistance
R\(_{comb}\)  Combined Convection and Radiation thermal resistance
R\(_{in}\)  Thermal conduction resistance of the inner wall
R\(_{out}\)  Thermal conduction resistance of the outer wall
R\(_{total}\)  Total thermal resistance of all layers of the wall
R\(_j\)  Thermal conduction resistance of material/wall “j”
n  Index number for the number of layers in the wall
q\(’\)  The heat flux
\(\dot{Q}\)  The rate of heat transfer through the whole wall
Q\(_\theta\)  Total thermal energy transmitted through the whole wall at time "\(\theta\)"
Q\(_{cond}\)  Rate of thermal energy transfer by conduction
Q\(_{conv}\)  Rate of thermal energy transfer by convection
\[ \dot{Q}_{rad} \] Rate of thermal energy transfer by radiation
\[ T_1, T_2 \] Temperatures of the exterior sides of the wall
\[ T_C \] Total cost of the selected sequence of materials
\[ T_H \] Total of heat blocking factors of the selected sequence of materials
\[ T_{in} \] Inner room temperature
\[ T_j \] The previous temperature of material/layer “j”
\[ T_{jp} \] The new temperature of material/layer “j”
\[ T_{out} \] Outer surrounding Sol-Air temperature
\[ T_{out, \theta - n \Delta t} \] Sol-Air temperature at time ( \( \theta - n \Delta t \) )
\[ T_{(p, m)} \] Temperature at position “m” at time “p”
\[ T_s \] The average temperature of the object
\[ T_{surr} \] The surrounding ambient temperature
\[ T_t \] Unit-less value used in optimization
\[ T(x,t) \] Temperature of the wall as a function of the position (x) and time(t)
\[ X_{jn} \] A variable that denotes the presence of arc j in the network, with “n” number of materials available in this arc.
\[ Z_C \] The optimum total cost
\[ Z_H \] The optimum total heat blocking factor
\[ Z_H \] Heat transfer matric coefficients
### Greek Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$\Delta T$</td>
<td>Change in temperature</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>Time interval used in transient heat transfer calculations</td>
</tr>
<tr>
<td>$\Delta x$</td>
<td>the thickness of the wall/layer</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Emissivity of the material</td>
</tr>
<tr>
<td>$\frac{\Delta R}{h_{co}}$</td>
<td>Long-wave radiation factor = $-3.9^\circ C$ for horizontal surfaces, $0^\circ C$ for vertical</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stefan Boltzmann constant</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Thermal diffusivity of the material</td>
</tr>
<tr>
<td>$\alpha_a$</td>
<td>Radiation absorptivity of the material</td>
</tr>
<tr>
<td>$h_{co}$</td>
<td>Surface color factor = 0.026 for light colors, 0.052 for dark</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density of the material</td>
</tr>
<tr>
<td>$\theta - n \Delta t$</td>
<td>Total heat flux through the whole wall at time ($\theta - n \Delta t$)</td>
</tr>
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### Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
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<tr>
<td>ASHRAE</td>
<td>American society of heating, refrigerating and air-conditioning engineers</td>
</tr>
<tr>
<td>CCN1</td>
<td>Cellular concrete-1</td>
</tr>
<tr>
<td>CCN2</td>
<td>Cellular concrete-2</td>
</tr>
<tr>
<td>CLTD</td>
<td>Cooling load temperature difference</td>
</tr>
<tr>
<td>FDM</td>
<td>Finite difference method</td>
</tr>
<tr>
<td>GBD</td>
<td>Gypsum board</td>
</tr>
<tr>
<td>HBF</td>
<td>Heat blocking factor</td>
</tr>
<tr>
<td>ICC</td>
<td>International code council</td>
</tr>
<tr>
<td>MRB</td>
<td>Marble</td>
</tr>
<tr>
<td>MWL</td>
<td>Mineral wool</td>
</tr>
<tr>
<td>SCB1</td>
<td>Sand cement blocks-1</td>
</tr>
<tr>
<td>SCB2</td>
<td>Sand cement blocks-2</td>
</tr>
<tr>
<td>SCP</td>
<td>Sand cement plaster</td>
</tr>
<tr>
<td>SLB</td>
<td>Sand lime brick</td>
</tr>
<tr>
<td>SPM</td>
<td>Shortest path method</td>
</tr>
<tr>
<td>TFM</td>
<td>Transfer function method</td>
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Chapter 1 Introduction

1.1. Background

Walls of buildings are constructed with a specific goal or combination of goals, as being bearable, protecting the inside atmosphere from being affected by the outer weather conditions, acting as a sound insulator to prevent the outside noise getting inside, acting as fire insulator to attenuate the spread of fire through the building, acting as water insulator to prevent the water of the rain getting inside the building, improving the building appearance and being of reasonable cost. All these goals are to be considered before constructing any wall. People in historic ages had their own standards of constructing walls based on limited types of materials and minimal considerations that have been enhanced by time to achieve optimum design of walls, providing the required internal environment of the building [1].

1.2. Walls construction and rules

1.2.1. Types of traditional materials and standards used

Buildings and foundations are the main characters that define different nations and their different eras of time. The walls construction relied mainly on the location of the building, its function, as being residential, commercial, industrial or military, and the features or ornamentation required on the walls. Walls were first constructed of bricks of clay manufactured in wood molds and were later manufactured in steel and iron molds, then the clay is fired to get its strength. Bricks were affordable and durable and that’s why they were dominant. These bricks were found in different shapes, sizes, colors and textures for various applications. Later on other materials were introduced to the walls construction process as granite, limestone, sandstone and marble. Wood was also used to give the building its unique character making use of its shaping capabilities and low weight. Other materials were also used as Stucco, which is a cement based material and Terracotta which is a glazed masonry product. Stalling of the building blocks is one of the main problems that can limit the durability of the wall, which is trapping water or humidity inside the block that leads to failure of the wall, so sealing the walls is necessary especially in roofs and any wall subjected to water or high humidity [1].
Buildings lose about 20-30% of its interior heated or cooled air to the surrounding through walls, while the rest is through doors, windows, machines, or people living inside the building. To save this portion of heat transfer through walls, insulation is required. For insulation, wood and foam were used between the main layers of the wall or in the cavities of the building blocks to help in achieving appropriate internal conditions by minimizing the heat getting inside in summer and heat escaping to the outside in winter [2].

Various materials with different designs of building blocks evolved by time with the main goal of achieving more durability and bearing characteristics while at the same time decreasing weight and cost. At the beginning, conventional walls standards and materials used were based on experience or trial and error. Later on, the building codes like “The International Code Council ICC” and “The International Building Code IBC” evolved based on technical analysis and theories. These codes specify the dimensions of the buildings, the thickness of the walls and the materials used in each wall [3],[4].

Later on, sustainability measures introduced to the buildings and some foundations like ASHRAE [5] evolved to help in achieving sustainable buildings by increasing the energy efficiency, enhancing the indoor air quality and decreasing the heating and cooling loads required in the buildings [5].

1.2.2. Bearing of walls

A non-load bearing wall is a wall that does not withstand any extra weight acting on it from the building, this means that it cannot support any load except its own. A load bearing wall is a wall that can withstand some loads from the building beside its own weight. Both types of walls can also be subjected to other exterior loads as wind loads and seismic loads.

The selected materials should be able to withstand the loads imposed by the building structure down to its foundation over the whole life of the building. The building is made of steel, concrete, wood and other building materials, beside the electrical and plumbing work inside the walls, so the wall should be designed to support all this weight. The stress analysis of the building specifies certain compression strength that should be afforded by the building materials to support the whole building with the people, furniture or any appliances inside the building. Joists and studs
are necessary to reinforce the walls to be able to withstand the loads over them. Non-bearing walls are using mainly internally for dividing the spaces within the building [1].

1.2.3. Thermal efficiency of walls and internal temperature

Construction walls are designed to isolate the internal environment in order not to be affected by the atmosphere outside the building. This is accomplished by minimizing the heat transfer through the walls because heat has to be kept where it is needed. Heat transfers from points of high temperature to points of low temperature. In the summer, heat is getting into the building while in winter, heat is escaping through the walls to the outer atmosphere. Both scenarios are not desirable by designers in order to eliminate weather conditioning costs whether in summer or winter. Thermally efficient walls will be necessary in some cases where internal environment has to be under control like in hospitals or in laboratories.

Thermal energy travels by conduction through the walls in either direction. The ability of the material to act as insulator to stop the inner room temperature from being affected by the outer temperature is determined by several factors, which are the thermal conductivity of the material (K) which determines the rate of heat transfer through the material. The less the value of the thermal conductivity, the more insulating is the wall. Moreover, the thermal capacity \((C_p)\) of the material which determines the amount of heat energy needed to raise the temperature of the material by 1 degree Celsius. The higher the value of the thermal capacity, the more the energy required to make the wall hot. Also the thickness \((L)\) of the wall affects the ability of the wall to act as thermal insulator. These objectives are satisfied by using heat insulators like wood, cork, or leaving air gaps in the walls to minimize heat transfer by conduction. These factors are combined to calculate the R-value of the wall, which is the thermal resistance that is used in comparing the thermal efficiency of different materials [6].

1.2.4. Compatibility of materials

One of the designers’ greatest challenges is to build the wall of materials that are compatible together. In certain cases, if a material is placed beside other materials, they affect each other negatively from the chemical and heat resistance perspective. At the most basic level, masonry cavity wall is constructed composed of two wythes of the same material, leaving air cavities with different dimensions between the wythes for better thermal performance, only a
single material was used. Later on, designers started looking for merging materials together while having minimal environmental footprint, being more energy efficient and Bearable. Then masonry was made of different materials that are compatible together as marble, granite, travertine, limestone, cast stone, or concrete blocks. Corrugated metal sheets are added as a kind of reinforcement. These materials are bounded together by mortar that is available of several types like Type- M and Type-N with different compression strength to build the finished wall. Stacking different materials together may be incompatible due to lack of binding between them, or different thermal expansion rates which may lead to cracks in extreme temperatures [1].

1.2.5. Costs

The cost of the material is one of the main aspects that affects the decision of material selection. The costs of the material include the initial cost of purchase, as some materials are more expensive than others due to their superior physical properties, thermal efficiency or durability. The costs also include the delivery costs of the material from the manufacturer to the construction site. Some factories do not have retailing centers, so the customers have to load the materials from the factories, which are located in remote areas; this adds to the delivery costs. The costs also include the maintenance and repairs of the building materials, as some materials need costly procedures and adhesives to maintain or repair. Some designers add the cost of air conditioning to the costs of the construction walls, as the thermal efficiency of the wall determines the air conditioning facilities required. So the designer has to compromise between the cost and the features of the materials used according to the application of the wall being constructed. For example, adding a layer of wood to the wall is very cost efficient in comparison with adding Aluminum layer instead. However, wood has low resistance to weather, moisture, warpage, and organic degradation and it is not as durable as aluminum [1].

1.2.6. Durability

The demand for high durability construction materials is increasing. The life time of the walls is the most important aspect of the building. The durability is affected by chemical and mechanical aspects. The wall should resist corrosion caused by lactic and acetic acids, and at the same time should have high compression and fatigue strengths to withstand the varying loads acting on it during its life time. Moreover, the decay resistance is one of the essential aspects in
durability, which is enhanced by heat and chemical treatment of the material. The building durability is affected by the exterior atmosphere and the climatic conditions surrounding the building like humidity, precipitation, wind, solar radiation. Concrete and cement are the most commonly used building materials due to their durability, however they are combined with other materials to achieve the other objectives of the designer. The durability of the wall is determined by the durability of the shortest life material used inside the wall. Gypsum, Stones, Metal reinforcement, timber and polymers have different life time [7].

1.2.7. Water resistance

Water resistance is the ability of the material to maintain its chemical and mechanical properties while being subjected to water for long time. The water may be in the form of rain or high humidity in the surrounding. The designers of the building walls are concerned by the softening coefficient of the material ($K_r$), that defines the degree of stability of mechanical properties of the material subjected to water.

\[ K_r \frac{F_b}{F_g} \]  

[8]  

$F_b$ is the compressive strength of the material in water saturation state (MPa), and $F_g$ is the compressive strength of the material in dry state (MPa). Designers seek high value of softening coefficient, which means that the material has high water resistance. The value of the softening coefficient required depends on the climate surrounding the building and the application of the building. Water resistant insulating layers can also be used to protect the material from being exposed to water, as it acts as a sealing technique to prevent water leakage [8].

1.2.8. Sustainability

Sustainability includes methods of production, ability to reuse, recycle or dispose of the material at the end of its life. For the material to be sustainable, the material selected should comply with the following two requirements. First, the material should not affect the environment by harming it or causing any pollution during any of the stages of its life cycle by studying the “Cradle to Grave” analysis of the material as shown in Figure 1. Second, the material should not affect the coming generations from using it by running out or becoming scarce. This means that the material should be renewable and plentiful. To evaluate the sustainability of any material and its impact on
the environment, life cycle assessment is constructed considering the extraction and manufacturing stages of the material, the sourcing, the installation stage, its performance and finally the waste treatment which is divided to disposal, recycling, or reusing the waste material. Limestone, Steel, Aluminum, Bricks and wood are considered sustainable due to their abundance and minimal effect on the environment [1].

![Figure 1: Sustainability of building materials](image)

1.3. Composite walls and advantages:

Composite structures are becoming more and more popular recently due to their superior features. A laminate composite wall is defined as a wall that is being formed of vertical layers of different materials or the same material but different grades. They have some major advantages as follows.

1.3.1. Advantages of composite walls:

The prominence of the composite walls is due to the fact that they can achieve many of the aspects previously mentioned in the previous section at the same time. For example, composite walls can afford high strength and stiffness, while at the same time affording high thermal efficiency and reasonable cost. All these aspects cannot be achieved by a wall of single material. One of the most common composite walls used is the concrete with the steel, because concrete has superior compression strength, while steel has high tension strength. Thus, combining both materials together can give out a wall with high tension and compression strengths, besides being relatively light in weight in comparison to materials with such physical properties. These materials
can be combined with one more layer or layers of another material that is poor conductor of heat like Foam, fiber glass or any type of plastics to achieve high thermal efficiency is addition to its high mechanical properties. Composite walls also provide another advantage in terms of speed of construction [8].

1.3.2. Construction concerns of composite walls:

Problems arise in composite walls when each material is considered individually and they are not studied as an assembly. Materials have to be compatible together in terms of chemical resistance to the atmospheric conditions and the thermal resistance in order to minimize the heat flow through the wall either into or from the room. The wall has to be bearable to all the loads acting on it during its life time. It is not necessary that all the materials in the composite structure to have high physical properties. The wall should not include two consecutive non-bearing layers, that each non-bearing layer has to be adjacent to at least one layer of bearing material. Moreover, the thickness of the wall has to comply with the standard thickness of walls according to “The International Building Code 2015” which are 3, 4, 6, and 8 inches for residential and light weight commercial buildings, while for heavy weight industrial building the most common thicknesses of walls used are 10 and 12 inches. So the number of layers used to form the composite wall is restricted by the maximum standard thickness allowed. The composite wall is formed either by stacking different materials walls beside each other (laminate composite) or by mixing the materials together before heating and injecting the composite material as a single layer. The laminate composite is the most prominent nowadays as no further processing is required, which means no further costs [5], [8].

1.4. Heat transfer through walls

Heat transfer is derived by difference in temperature between two points. Heat always moves according to temperature gradient, from point of high temperature to low temperature. If there is no difference in temperature there will be no transfer of heat energy, and this case is called “Thermal Equilibrium”. There are three means to transferring heat energy, which are conduction, convection and radiation [10].
1.4.1. Heat transfer mechanisms

Conduction is a way of transferring energy from more energetic particles to less energetic particles by interaction between them. Conduction in gases occurs due to the contact of moving molecules. For Non-metals, either in the liquid or solid states, conduction occurs throughout the molecules by the hitting of neighbor molecules and vibration. While in metals, conduction is at its highest rate due to the transfer of energy by the free moving electrons. The rate of heat transfer by conduction through a wall ($\dot{Q}_{\text{cond}}$) relies on the Thermal conductivity of the material of the wall (K), the surface area being studied (A), the thickness of the wall ($\Delta x$) and the temperature difference between the exterior surfaces of the wall ($\Delta T$) [10].

$$\dot{Q}_{\text{cond}} = K A \frac{\Delta T}{\Delta x} \quad [10] \quad (2)$$

Convection is a way of transferring energy between a solid surface and the surrounding liquid or gas. The molecules surrounding fluid are always in motion as the molecules with high kinetic energy have more separation which decreases the density of this region of the fluid, so it moves up and is replaced by cooler region. The moving molecules exchange heat energy with any nearby solid surface. Convection can be categorized as free convection or forced convection. Free convection happens due to the buoyance derived by the difference in density of different regions of the fluid, while forced convection occurs if the fluid is forced to flow in a certain direction by any external means as a fan or wind. Rate of heat transfer by convection ($\dot{Q}_{\text{conv}}$) is dependent on the convection heat transfer coefficient of the surface, that depends on the geometry of the surface, the fluid properties and its velocity. ($\dot{Q}_{\text{conv}}$) also depends on the surface area being studied (A), the surface temperature ($T_s$) and the surrounding fluid temperature ($T_{\text{surr}}$). The sign of the resulting value identifies the direction of heat transfer, either into or from the body [10].

$$\dot{Q}_{\text{conv}} = h_{\text{conv}} A (T_s - T_{\text{surr}}) \quad [10] \quad (3)$$

Radiation is different from conduction and convection, as it can transfer heat energy between two bodies that are not in contact, even if they are separated by vacuum. Radiation is the transfer of heat energy through infra-red radiations, which is a type of electromagnetic waves as a result of the changes of the electronic configuration of the atoms of the object. Radiation is fastest means of transferring heat energy; it is the way of transferring heat energy from the Sun to the
Earth. Anybody at a temperature above absolute zero emits heat radiation. Radiation is a volumetric phenomenon that occurs throughout the whole object, however for opaque objects it is considered to happen at the surface only as the radiation from the inner parts of the objects will not be able to reach the surface. The net rate of heat radiation transfer between the object and the surrounding medium ($\dot{Q}_{\text{rad}}$) is dependent on the emissivity of the material ($\varepsilon$), the surface area being studied ($A$), the object’s temperature ($T_s$) and the surrounding temperature ($T_{\text{surr}}$) \[ 10 \].

\[
\dot{Q}_{\text{rad}} = \varepsilon \ 6 \ A \ (T_s^4 - T_{\text{surr}}^4) \tag{4}
\]

where 6 is the Stefan Boltzmann constant, $6 = 5.67 \times 10^{-8}$ W/m$^4$

1.4.2. Thermal resistance concept:

Conduction heat transfer equation (2) can be rearranged to include a new factor which is the thermal conduction resistance ($R_{\text{cond}}$) \[ 10 \].

\[
\dot{Q} = \frac{1}{R_{\text{cond}}} \ (T_1 - T_2) \tag{5}
\]

\[
R_{\text{cond}} = \frac{\Delta x}{KA} \tag{6}
\]

The same applies to convection and radiation; the heat transfer equation is rearranged to include a new factor which is the thermal convection and radiation resistance ($R_{\text{comb}}$). This factor depends on the convection and radiation heat transfer coefficients $h_{\text{conv}}$ and $h_{\text{rad}}$ \[ 10 \].

\[
\dot{Q} = \frac{1}{R_{\text{comb}}} \ (T_1 - T_2) \tag{7}
\]

\[
R_{\text{comb}} = \frac{1}{hA} \tag{8}
\]

\[
h = h_{\text{conv}} + h_{\text{rad}} \tag{9}
\]

For steady state heat transfer, where the temperatures of the internal and external sides of the wall are kept the same all the time, the wall can be modeled as an electric resistor as shown in Figure 2 \[ 10 \].
In case more than one mode of heat transfer or more layers of walls stacked by each other available, the “R” is replaced by “R_{total}”. R_{total} is the sum of the thermal resistance of all the layers present between the temperatures of any two points T_1 and T_2 [10].

1.5. Relation between external temperature, location and orientation of the building:

One of the main objectives of the design of the construction wall is to achieve high thermal efficiency, which means minimal amount of heat energy being transferred through the wall in either direction to reach the required internal temperature of the room throughout the whole year seasons, and this depends on the climate that the building is facing that depends on the location and the orientation of the building. The main aspect of the climate that affects heat flow is the ambient temperature of the atmosphere.

As the atmosphere of the Earth is not equally heated, so different regions of the Earth have different temperatures. The atmospheric temperature relies on many factors such as the Altitude of the location, the latitude, the topography, the nearby water, the vegetation of the land and the prevailing winds [11].

The latitude is how far the location is from the equator. The further the location from the equator, the lower the temperature it has. This is due to the fact that the Earth’s surface is a curve, so the sun rays has to move larger distance to reach the points with higher latitudes so more energy is lost and the temperature of these points become lower, as shown in Figure 3. This explains why the temperature near the poles is lower [11].
The altitude defines how high is the location above the surface of the sea. The higher the altitude, the lower the atmospheric temperature. For every 100 m rise in altitude, the atmospheric temperature decreases by 1°C. The surface of the earth absorbs heat energy from the sun, that diffuses to the nearby atmosphere, later on by convection some of this heat is transferred to the upper layers. Also as the higher levels of atmosphere has less dense and less water vapor which absorbs less heat energy [11].

Oceans and seas take more time to heat up or cool down than land due to the high thermal capacity of water, that is why the coastal regions will have lower temperature in summer and higher temperature in winter in comparison with inland regions at the same altitude and latitude [11].

The prevailing wind is the wind that approaches the location in the most common direction. The wind may introduce hot or cold air to the location according to the place it is coming from. Topographic barriers such as mountains and hills affect the flow of the wind, which affects the atmospheric temperature [11]. The wind also affects the vegetation of the land which also impacts the surface temperature. Short ordered plants increase the temperature due to the excessive water vapor produced by the plants that causes greenhouse effect, while tall and complex structures as forests decrease the temperature as they obstruct the sun rays from reaching the ground [11]. All the previously stated factors combined justify the differences in temperature in the different locations worldwide as shown in Figure 4.
The orientation of the building is the second factor that affects the heat flow through the walls of the building. The main process of gaining heat energy by the building’s walls or roof is through solar irradiance (radiation), which is the power per unit area received from the sun. Solar radiation immediately affects the temperature of surface it is hitting. Solar radiation is divided into three types, which are direct radiations moving in straight lines from the sun to the building. Diffuse radiation that are scattered by hitting the molecules of the atmosphere changing its direction to reach the building, and reflected radiation that is reflected backwards from non-atmospheric things such as the ground to reach the building [5].

The orientation of the wall receiving the sun rays affects the angle of incidence of the rays, the amount of shading of the wall and the time of exposure of the surface to the sun during the day. All these factors affect the temperature of the wall. Horizontal walls/ roofs receive long wave radiations from the sky only, while vertical walls receive long wave radiation from the sky and reflections from the ground and surrounding buildings (reflected radiation). The orientation of the building has to be considered while calculating the heat flow through the walls, as it affects the CLTD value (cooling load temperature difference) used in cooling load calculations [5].
1.6. Typical day of the year:

While calculating the heat flow through the walls of the building, the outer temperature of the atmosphere has to be defined as it affects the rate of heat flow into or out of the walls. However, the temperature of any location is variable throughout the whole year. It reaches the minimum temperature at a certain time in winter and the maximum temperature at a certain time in summer. Considering one of these peak values in calculating the heat flow through the walls will not be representative of the whole year and the error will be large at certain times of the year. Thus the temperature of the “typical day of the year” should be considered, that shows the average temperature of the location throughout the whole year. The typical day is represented in hourly temperature profile of the 24 hours that can be used to calculate the average heat transfer through the walls [5].

1.7. Sol-Air temperature

The transfer of heat through the layers of the composite wall does not rely only on the difference between the inner room temperature and the outer surrounding temperature. The outer surface of the wall is also subjected to incident heat radiations from the sun beside the surrounding temperature; this means that heat is transferred into or out of the outer exposed surface by convection and radiation. To compensate for the amount of radiation incident on the wall, Sol-Air temperature is used rather than the surrounding ambient temperature. Sol-Air temperature is a value of the outdoor temperature that gives the same heat transfer in the absence of any heat radiations as the combination of convection and radiation heat transfer at any time. It compensates for the total solar radiation, radiant exchange with the sky and other outdoor surroundings. The Sol-Air temperature is used in the cooling or heating loads calculations of buildings [5].
Chapter 2 Literature Review

2.1. Previous research

Buildings construction sector represents a considerable part in total energy consumption and CO\(_2\) emissions worldwide. The building industry consumes 40% of the whole energy consumption and produces 36% of the CO\(_2\) produced worldwide. However, with some considerations, the buildings sector can represent a source of energy savings as 75% of the buildings worldwide are described as being inefficient in terms of heat transfer through the walls of the building which leads to high heating or cooling loads requirements [14].

One of the main aspects of constructing a sustainable building is the use of sustainable materials. Selecting materials with optimum functional properties and being sustainable at the same time is a complex process. Several studies examined new materials that can afford minimal thermal transport through them and save energy, without compromising the strength and stability of the materials [15]. Most of the research studies are concerned with controlling the thermal properties of the wall through controlling the insulation material or thickness, or by controlling the air tightness in in the building envelope, which is effective in the heating season, while being a drawback in the cooling season as it causes overheating of the room. Later on, it was discovered that controlling the building materials of the wall can play a considerable role in controlling the thermal properties of the wall itself, which in turn defines the degree of sustainability of the building [8]. Pezeshki et. al. [8] studied the properties of different building materials in his research as Aluminum, Brick, Ceramic, Cement, Concrete, Glass, Marble, Plaster, and Granite. Several characteristics of these materials have been studied as thermal conductivity, heat flux and emissivity constant effects. These properties affect the heat transfer through the building wall, which in turn affects the cooling and heating costs required throughout the whole life of the building. Moreover, this research studied other specifications of these materials as Density, Porosity, Solidity, fill rate, water resistance and mechanical strength of each material [8].

Another study showed that the thermal properties of the material are not only dependent on its type, but also depend on the particle shape, size and micro structure. That lead to a new conclusion that the thermal properties of the same material can be altered by controlling its processing techniques [16]. Leaving cavities and porosities between the molecules of the material
affects the rate of heat transfer through it by introducing more means of heat transfer between the molecules other than just conduction, as convection and radiation also occur between the molecules of the material as shown in Figure 5 [16].

Figure 5: modes of heat transfer through porous materials [16]

Heat transfer through the walls and ceilings of buildings has been studied in numerous previous studies, especially for the sake of heating or cooling loads calculations of the buildings. The three modes of heat transfer (conduction, convection and radiation) were examined in several studies. Pezeshki et.al [8] showed the necessity of considering the heat transfer by Conduction through the walls, showing different materials with small value of heat conduction coefficient (k). Then the study showed the effect of considering the Convection and Radiation through the exterior surface of the wall, as there is Convection between the surrounding air and the surface of the wall and Heat Radiation from the Sun (either direct or diffusion) incident on the wall, or due to reflection from the surrounding objects [8]. Cavazzuti et. al [17] studied the heat transfer by conduction through composite walls, by forming an algorithm to solve the steady state heat transfer. This study connects the network of thermal resistors of the layers in the composite wall, showing the series and parallel connections and considering how the thermal properties of the materials change with temperature.
Loveday et al. [18] presented in his study the necessity of considering the heat transfer to the exterior wall of the building by convection from the surrounding, by experimenting an eight storey building. He explained the way of calculating the convection heat transfer coefficient \( h_{\text{conv}} \), showing how this coefficient is dependent on the external wind speed and the geometry of the wall. Evangelisti et. al [19] explained the way of calculating the convection and radiation actual coefficients by examining several case studies and comparing different techniques of comparing the thermal coefficients. The value of the convection coefficient \( h_{\text{conv}} \) is calculated by any of the standards as shown in figure 6 [19]. While the radiation heat transfer coefficient is calculated by equation (10). The values calculated by these methods is compared with the experimental results measured on site, that showed minor deviation [19].

\[
h_{\text{rad}} = \varepsilon 6 \left( T_s^2 - T_{\text{surr}}^2 \right) \left( T_s - T_{\text{surr}} \right) \tag{10} \]

![Figure 6: Convection heat transfer \( h_{\text{conv}} \) by different theories [19]](image)

The advantages of composite walls rather than traditional walls of single material are listed in several research papers [8], [20]. The most prominent advantage of using composite structures is the ability to achieve combination of superior characteristics at the same time, like lightweight, high strength, low costs [8]. Nikbakt et.al reviewed several studies related to the optimization of composite structures in order to achieve different objectives, like minimum weight of the wall, minimum cost, maximum strength, or minimum deflection. Different composite structures were
studied, composite beams, composite plates, composite shells and other composite structures [20]. Other theoretical and experimental studies proved that using composite structures allows achieving superior thermal properties in terms of low thermal conductivity (less than \(0.5 \frac{W}{m.k}\)) and high insulation properties, besides being mechanically stable [8].

The heat transfer through the walls of a building is considered as one dimensional situation rather than multi-dimensional as the heat transfer through the thickness of the wall is much faster than that in the two other dimensions of the wall, this is due to the fact that the thickness is much smaller than the length and the width of the wall [21]. Even for complex structures, an equivalent U-factor for the structure is determined using” Equivalent Homogeneous Layer Method”; afterwards the problem is solved as one dimensional [21].

The heat transfer through the walls of a building was considered as a steady state case in several studies, which means that the inner and outer temperatures of the wall are constant all the time and rate of heat transfer through the wall is constant [17]. The limitation of this consideration is that it calculates the heat transfer by considering the thermal resistance of the layers of the wall only without considering the amount of heat energy absorbed by the material itself to become heated, which is called the heat storage capability. Later on, the building walls were modeled as transient heat transfer cases [14]. Francesco Leccesea [14] studied the effect of the variation of the external surrounding temperature and the different materials of the layers on the internal temperature and the heat transfer through the walls of the building. This study applied the heat transfer matrix method to calculate the heat flux through the walls considering combinations of different materials. The schematic representation of one wall the heat energy gained by the wall \((q_i)\) and the heat energy lost by the wall \((q_e)\) is shown in Figure5, where the \(Z_n\) is the heat transfer matrix coefficient. The approach takes into consideration the heat capacity, thermal resistance \((R_i, R_e)\) and the thickness of the wall \((\Delta x)\). In addition to the external temperatures \((T_i \text{and } T_e)\).
Further studies stated the effect of the sequence of the layers on the heat transfer through the wall\cite{22}. Kossecka et.al studied six different configurations for the layers of the wall, placing the insulation in different position in each configuration. The results showed that the best thermal performance is achieved when the material with high heat capacitance is placed at the interior side of the wall. However, this is dependent on the climate surrounding the building and the season of the year, so Leccese \cite{14} suggested that both sides of the wall should include the materials with the largest heat capacitance.

Selecting the materials used in the walls of the buildings is dependent on many factors, as mechanical properties, heat transfer, durability and cost \cite{1}. Abdelghani et.al \cite{23} studied a way of selecting the optimum sequence of materials used in building a composite wall from the cost and heat transfer points of view. The study considereed the bearing condition, that the wall should not include two consecutive non-bearing layers. Abdelghani et.al \cite{23} used a Shortest Path Method (SPM), representing the wall as a network with several paths to form the wall with two objectives, which are minimum cost and minimum heat transfer through the wall. However, this study applied the steady state heat transfer scenario through the walls. The steady state heat transfer model assumes that the temperature is constant at both sides of the wall and this is not realistic due to the daily and seasonal fluctuations of the surrounding temperature. Moreover, only heat transfer by conduction is considered in this study, neglecting the convection and radiation processes at the outer layers of the wall. Moreover, the effect of the sequence of the materials is not considered.
2.2. Scope of work

Several research papers showed different techniques for selecting the optimum composite wall layers based on a single objectives as thermal transfer or the mechanical properties of the wall. The main goal of this research is to identify the most thermally efficient composite building wall considering achieving minimum cost and heat transfer through it; given its location, orientation and available building materials. It is required to model the heat flow from the outdoors through the composite walls forming the building that consists of multiple layers based on transient heat transfer conditions. Such modeling must take into account the external conditions as the ambient temperature and the amount of radiations incident from the sun on the walls based on the location of the building and the orientation of each wall.

2.3. Research objective

The main goal of this research is to identify the most thermally efficient composite building wall considering two objectives, which are achieving minimum cost and heat transfer through the wall; given its location, orientation and available building materials in the market with their properties.
Chapter 3 Methodology and Approach

Each wall in any building can be constructed of single layer or multiple layers in composite (laminates) form. There are two objectives to be accomplished here which are achieving the minimum cost of the wall and the minimum thermal transfer through the wall to keep the inner room temperature stable, by selecting the number of layers and the corresponding material of each layer. Moreover, the wall should be able to withstand the loads acting on it, each non-bearing material should not be adjacent to another non-bearing material “the bearing condition” [23]. The final thickness of the wall should be within the desired limits stated by the designer or the standard limits stated by ASHRAE [5] (from 4 in to 12 in) [5]. The approach starts with drawing the Networks of the different number of layers for the wall considering the bearing condition, followed by the optimization process considering both the minimum cost and heat transfer objectives. Afterwards, the optimized sequence of materials for each number of layers will be used to calculate the transient heat transfer through it. Finally, the different alternatives will be presented showing the optimum sequence of materials for each number of layers with the corresponding cost and heat transfer through it. Figure 8 shows a flow chart representing the sequence of the methodology followed in this study.
Figure 8: Steps of the approach followed in this study
3.1. Optimization model

To get the optimized sequence of materials considering the cost and heat considerations, multi-criteria applying Pareto optimization technique will be implemented. At the beginning, each objective will be considered separately using two separate linear models, to get the optimum sequence of materials regarding the cost neglecting the heat transfer, then getting the optimum heat transfer neglecting the cost. Finally, a third model will be constructed seeking the minimal deviation from the first two objective functions.

The shortest path method (SPM) [23] is implemented to get the optimum sequence of layers. A network is drawn with M nodes indexed by i and N arcs indexed by j. Each complete path should start from \( i = 1 \) which is the “Source” node and end at \( i = N \) which is the “Sink” node, passing through certain number of arcs (Ex: \( j = 1,4,6 \)). A variable \( X_j \) denotes the presence of arc \( j \) and its corresponding cost is \( C_j \).

Before constructing the model, the network should be drawn considering the number of materials required (1 or 2 or 3 or 4) and the bearing condition. The optimization models will be run for each number of layers at a time, to get the optimum wall construction in case of one or two or three or four layers separately then the four results will be presented finally for the decision maker to select the best sequence according to the budget limits or the heat transfer limitations.

In the network, each arrow represents a single material. For example to represent 4 different materials that can be used as the first layer it can be modeled as shown. However, to make it easier in drawing and to save space, this group of materials is represented by a single arrow labeled \( X_{jn} \).
For 1 layer:

For 2 layers:

For 3 layers:

For 4 layers:
3.2. Minimal cost model

The objective function of the model is to: Minimize total cost (Zc)

\[ \text{Min} \sum_j C_j X_j \]

Subject to:

1) The sum of all arcs coming out of the first node should be equal to 1.

\[ \sum_{\text{out of first node}} X_j = 1 \]

2) The sum of all arcs going into the last node should be equal to 1.

\[ \sum_{\text{into last node}} X_j = 1 \]

3) The sum of all arcs going into the any node “i” should be equal to the sum of all arcs coming out of it, for all nodes (i = 1 to m)

\[ \sum_{\text{into node i}} X_j = \sum_{\text{out of node i}} X_j \]

4) The thickness of the whole wall should be within the required limits of desirable thickness.

\[ L_{\text{min}} \leq \sum L_j X_j \leq L_{\text{max}} \]

5) All values of \( X_j \) are binary numbers, as the flow of the arc is either present or not, so it has to be either 0 or 1.

3.3. Minimal heat transfer model

The dynamic behavior of the temperature of the wall T(x,t) is dependent on both the position of the point under consideration (x) and the time (t). The change in temperature is given by the equation:

\[ \frac{dT(x,t)}{dt} = \frac{k}{\rho C_p} \frac{d^2T(x,t)}{dx^2} \]  \[\text{[6]}\]  \[\text{[11]}\]

\[ \alpha = \frac{k}{\rho C_p} \]  \[\text{[10]}\]  \[\text{[12]}\]
The Thermal diffusivity (\(\alpha\)) is a measure of how quickly heat can be transferred through the material. It is used in case of transient heat transfer situations, as the material does not just pass the heat through it, but it also absorbs some of this heat to be heated. Equation (11) is implemented into the energy balance equation (13) of a single layer wall as shown in Figure 9.

\[
E_{in} - E_{out} = E_{stored} \quad \text{[6]} \tag{13}
\]

\[
KA \frac{T_{(p,m-1)} - T_{(p,m)}}{L} + KA \frac{T_{(p,m+1)} - T_{(p,m)}}{L} = \rho A L C_p \frac{T_{(p+1,m)} - T_{(p,m)}}{\Delta t} \quad \text{[6]} \tag{14}
\]

Figure 9: Transient heat transfer through a single layer

Minimizing the heat transfer can be achieved by minimizing the change in temperature of the internal surface of the wall \((T_{(p+1,m)} - T_{(p,m)})\), which can be done by maximizing the heat blockage factor \(\left(\frac{\rho C_p L^2}{k}\right)\) of the materials used in constructing the wall. As the value of the heat blockage factor (HBF) increases, the heat transfer through the wall decreases. The heat blockage factor for wall j will be represented by the symbol HBF\(_j\). The heat blockage factor is also stated by Leccese [14] while implementing the heat transfer matrix method. As the wall heat transfer coefficients (\(Z_n\)) includes the heat blockage factor stated above.

The objective function of the model is to: Maximize the HBF of the layers (\(Z_H\))

\[
\text{Max} \sum_j HBF_j X_j
\]

Subject to:

1) The sum of all arcs coming out of the first node should be equal to 1.
The sum of all arcs going into the last node should be equal to 1.

\[ \sum_{\text{into last node}} X_j = 1 \]

3) The sum of all arcs going into any node “i” should be equal to the sum of all arcs coming out of it, for all nodes (i = 1 to m)

\[ \sum_{\text{into node } i} X_j = \sum_{\text{out of node } i} X_j \]

4) The thickness of the whole wall should be within the required limits of desirable thickness.

\[ L_{\text{min}} \leq \sum_j L_j X_j \leq L_{\text{max}} \]

5) All values of X_j are binary numbers, as the flow of the arc is either present or not.

3.4. The dual objectives model

The goal of this model is to get the optimum sequence of layers that will achieve the two previous objectives at the same time which is the minimal cost and the minimal heat transfer through the walls.

The objective function of the model is to: Minimize the deviation from the optimum cost \((Z_C)\) and the optimum HBF \((Z_H)\) using a new symbol \((T_t)\).

\[ \text{Min} \ T_t \]

Subject to:

1) The sum of all arcs coming out of the first node should be equal to 1.

\[ \sum_{\text{out of first node}} X_j = 1 \]

2) The sum of all arcs going into the last node should be equal to 1.
\[ \sum_{\text{into last node}} X_j = 1 \]

3) The sum of all arcs going into the any node I should be equal to the sum of all arcs coming out of it, for all nodes \((i = 1 \text{ to } m)\)

\[ \sum_{\text{into node } i} X_j = \sum_{\text{out of node } i} X_j \]

4) All values of \(X_j\) are binary numbers, as the flow of the arc is either present or not.

5) Cost constraint:

\[ T_c = \sum_j C_j \cdot X_j \]

6) Cost deviation from the optimum value:

\[ T_c - Z_c = O_c - B_c \]

\((O_c: \text{value over the optimum cost}, B_c: \text{value below the optimum cost})\)

7) Heat constraint:

\[ T_H = \sum_j H_{BF_j} \cdot X_j \]

8) Heat deviation from the optimum value:

\[ Z_H - T_H = O_H - B_H \]

\((O_H: \text{value over the optimum heat blockage}, B_H: \text{value below the optimum heat blockage})\)

9) Cost deviation as factor of the optimum cost value:

\[ \frac{O_c}{Z_c} \leq T_t \]

10) Heat blockage deviation as factor of the optimum heat blockage value:

\[ \frac{O_H}{Z_H} \leq T_t \]

\(T_t\) “is a unit-less value used to minimize the left-hand side of the equation.

11) The thickness of the whole wall should be within the required limits of desirable thickness

\[ L_{\text{min}} \leq \sum_j L_j \cdot X_j \leq L_{\text{max}} \]

12) All variables have non-negative values:
\[ X_1, X_2, \ldots, X_j, O_H, B_H, T_H, O_C, B_C, T_C, T_t \geq 0 \]

3.5. Thermal model

The heat transfer through the wall is calculated using MATLAB [24] function as follows:

3.5.1. Input parameters

\( C_{p1, \ldots, j} \) “Specific heat capacity of the materials used”

\( \rho_{1, 2, \ldots, j} \) “Density of the materials used”

\( K_{1, 2, \ldots, j} \) “Coefficient of thermal conductivity of the materials used”

\( T_{\text{in}} \): the inner room temperature

\( T_{\text{out}} \): array of 24 values of the outside sol-air temperature

\( L_{1,2,\ldots,j} \) “Half the thickness of each material (layer)

\( h_i \): coefficient of heat convection of the inner wall.

\( h_o \): coefficient of heat convection of the outer wall.

\( \Delta t \): time interval between calculations (\( \Delta t \))

\( T_s \): initial temperature of the whole wall

\( R_j \): Thermal conduction resistance

\( R_{\text{in}}, R_{\text{out}} \): Thermal convection resistance at the inner and outer walls, respectively

\( T_j \): the previous temperature of layer \( j \).

\( T_{jp} \): the new temperature of layer \( j \).

\( A \): Surface area considered (here \( A = 1 \text{ ft}^2 \))

\( \hat{Q} \): Total Heat Flux through the wall
3.5.2. Calculating the Sol-Air temperature

\[ T_{\text{out}} = T_{\text{surr}} + I_t \frac{\alpha_a}{h_c} - \varepsilon \frac{\Delta R}{h_c} \]  

\[ \alpha_a \]: radiation absorptivity of the material

\[ I_t \]: global solar radiation (irradiance) = 1.15 (SHGF), where SHGF is the solar heat gain factor provided by ASHRAE, chapter 29, table 15 [5].

\[ \frac{\alpha_a}{h_c} \]: surface color factor = 0.026 for light colors, 0.052 for dark

\[ \varepsilon \frac{\Delta R}{h_c} \]: long-wave radiation factor = -3.9°C for horizontal surfaces, 0°C for vertical

3.5.3. Calculating the thermal resistance of each layer

The Thermal resistance of each layer, as shown in Figure 10, can be calculated by the following equations.

\[ R_j = 0.5 \frac{L_j}{K_j} \]  

\[ R_{in} = \frac{1}{Ah_i} \]  

\[ R_{out} = \frac{1}{Ah_o} \]
3.5.4. Crank-Nicolson method iterations:

Crank-Nicolson is a finite difference method used for solving transient heat equations and other partial differential equations. This method is implicit in time, which means that the results are always numerically stable without any time interval ($\Delta t$) limitations. The heat equation for the first layer is shown (19).

\[
\frac{T_{in} - T_1}{R_{in} + R_1} + \frac{T_2 - T_1}{R_2 + R_3} = \frac{\rho_1 A_1 L_1 C_p_1}{dt} \left( T_{1p} - T_1 \right) \tag{6} \tag{19}
\]

This equation is repeated for the rest of the layers until the last layer is reached.

\[
\frac{T_{(j-1)} - T_j}{R_{(j-1)} + R_j} + \frac{T_{out} - T_j}{R_{out} + R_j} = \frac{\rho_j A_j L_j C_p_j}{dt} \left( T_{jp} - T_j \right) \tag{6} \tag{20}
\]

The factor $\frac{\rho_j A_j L_j C_p_j}{dt}$ can be represented by the symbol CM.

\[
CM_j = \frac{\rho_j A_j L_j C_p_j}{dt} \tag{6} \tag{21}
\]

The iterations continue for certain number of iterations per hour according to the selected time interval, to get the hourly inner temperature of layer 1 ($T_1$). Using the value of $T_1$, the heat flux (rate of heat transfer, $\dot{Q}$) to the room can be calculated.

\[
\dot{Q} = \frac{T_1 - T_{in}}{R_{in} + R_1} \tag{6} \tag{22}
\]

3.5.5. Approach verification

The results of the finite difference method (FDM) or Crank–Nicolson constructed on MATLAB [24] can be verified by comparing with the transfer function method (TFM) in ASHRAE [5]. The heat transfer through a wall or a roof consisting of one or more layers at hour $\theta$ can be calculated, using MATLAB [24], by equation (23). The results of the FDM and TFM methods are summarized in Table 1:

\[
Q_\theta = A \left[ \sum_{n=0}^{\infty} b_n T_{(out, \theta - n \Delta t)} - \sum_{n=1}^{\infty} \frac{d_n \dot{Q} (\theta - n \Delta t)}{A} - T_{in} \sum_{n=0}^{\infty} c_n \right] \tag{5} \tag{23}
\]

where:
A is the surface area (A = 1 ft²)

n is an index number indicating the number of walls

\( b_n, c_n, d_n \) are the conduction transfer function coefficients (Tables 13, 14, 18, 19; chapter 21; 1997 ASHRAE Fundamentals Handbook) [5]

\( \Delta t \) is the time interval

\( T_{(\text{out}, \theta - n\Delta t)} \) is the sol-air temperature at time \( (\theta - n\Delta t) \), it is an array of 28 values of temperature (starting by the last 4 hours of the previous day followed by the 24 values of the day)

\( T_{\text{in}} \) is a constant indoor room temperature

\( \dot{Q}_{(\theta - n\Delta t)} \) is assumed to be zero at the beginning, as the heat transfer of the previous time is not calculated yet. However, the effect of this assumption becomes negligible as the calculation is repeated for the successive 24 hours.

Table 1, shows that the results of both methods are approximately the same. The difference is less than 3%. The inner wall temperature at each hour throughout the whole day is compared, and the results of the two methods are listed and plotted on the same graph, as shown in Figure 11.

<table>
<thead>
<tr>
<th></th>
<th>Finite Difference Method (FDM)</th>
<th>Transfer Function Method (TFM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daily total heat transfer</td>
<td>27.4377 ( \frac{Btu}{ft^2} )</td>
<td>26.7130 ( \frac{Btu}{ft^2} )</td>
</tr>
<tr>
<td>Inner wall final temperature at the end of the day</td>
<td>75.9704°F</td>
<td>76.0472°F</td>
</tr>
</tbody>
</table>
The HBF shows the ability of the single material layer to resist heat transfer through it considering the heat conductivity of the material, its thermal capacity, density and thickness. As the HBF increases, the heat transfer through the wall decreases. To verify the HBF concept, ten construction materials, shown in Table 2, are tested using the transient heat transfer model for single wall. The results of the temperature variation throughout the whole day and the total heat transfer are as shown in Tables 2 and 3. Considering the inner room temperature $T_{in} = 75 \degree F$.

### Table 2: Hourly inner wall temperature (in °F) for single layer wall, for different materials

<table>
<thead>
<tr>
<th>Day</th>
<th>$T_{out}$</th>
<th>SCB1</th>
<th>SCB2</th>
<th>CCN1</th>
<th>CCN2</th>
<th>SLB</th>
<th>WOD</th>
<th>GBD</th>
<th>SCP</th>
<th>MWL</th>
<th>MRB</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>72.0</td>
<td>76.44</td>
<td>76.13</td>
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<td>CCN1</td>
<td>CCN2</td>
<td>SLB</td>
<td>WOD</td>
<td>GBD</td>
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<td>MRB</td>
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<td>83.26</td>
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</tbody>
</table>

Where SCB2 is sand cement blocks-1, CCN1 is cellular concrete-1, CCN2 is cellular concrete-2, SLB is sand lime brick, WOD is wood, GBD is gypsum board, SCP is sand cement plaster, MWL is mineral wool and MRB is marble.

**Table 3: HBF and total daily energy flux along the whole day for different materials**

<table>
<thead>
<tr>
<th></th>
<th>SCB1</th>
<th>SCB2</th>
<th>CCN1</th>
<th>CCN2</th>
<th>SLB</th>
<th>WOD</th>
<th>GBD</th>
<th>SCP</th>
<th>MWL</th>
<th>MRB</th>
</tr>
</thead>
<tbody>
<tr>
<td>HBF</td>
<td>1.03</td>
<td>9.65</td>
<td>0.81</td>
<td>13.23</td>
<td>2.3</td>
<td>0.15</td>
<td>0.01</td>
<td>0.02</td>
<td>4.06</td>
<td>0.04</td>
</tr>
<tr>
<td>$q''$ ($\frac{Btu}{ft^2}$)</td>
<td>428.62</td>
<td>244.97</td>
<td>434.48</td>
<td>198.47</td>
<td>387.57</td>
<td>481.89</td>
<td>510.02</td>
<td>509.93</td>
<td>275.83</td>
<td>508.06</td>
</tr>
</tbody>
</table>
Chapter 4 Results and Discussion:

As stated before in section 2.2, the aim of this research is to help the construction designer to decide which material or sequence of materials to be used in building a composite laminate wall in order to achieve optimum cost and heat transfer objectives, while considering the bearing ability of the wall and the thickness of the final wall to be within the required limits. After studying the most common materials used in construction in the market, it was found that there are different materials being used with different properties. Ten materials are used to decide the optimum wall sequence following the approach of this research. The materials selected are divided into bearing and non-bearing. The bearing materials are Sand Cement Blocks-1, Sand Cement Blocks-2, Cellular Concrete-1, Cellular Concrete-2 and Sand Lime Brick. While the non-bearing materials are Wood, Gypsum Board, Sand Cement Plaster, Sand Cement Plaster and Mineral Wool. The properties of the materials are listed in Table 4.

<table>
<thead>
<tr>
<th>Bearing Materials</th>
<th>Code</th>
<th>(C_p) (Btu/Ib-°F)</th>
<th>(K) (Btu/hr-ft-°F)</th>
<th>Thickness (ft)</th>
<th>(\rho) (Ib/ft(^3))</th>
<th>Cost ($/ft(^2))</th>
<th>HBF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand Cement Blocks-1</td>
<td>SCB1</td>
<td>0.2</td>
<td>1.9</td>
<td>0.33</td>
<td>90</td>
<td>1.1</td>
<td>1.03</td>
</tr>
<tr>
<td>Sand Cement Blocks-2</td>
<td>SCB2</td>
<td>0.2</td>
<td>0.93</td>
<td>0.67</td>
<td>100</td>
<td>1.55</td>
<td>9.65</td>
</tr>
<tr>
<td>Cellular Concrete-1</td>
<td>CCN1</td>
<td>0.2</td>
<td>1</td>
<td>0.17</td>
<td>140</td>
<td>1.15</td>
<td>0.81</td>
</tr>
<tr>
<td>Cellular Concrete-2</td>
<td>CCN2</td>
<td>0.2</td>
<td>0.4</td>
<td>0.42</td>
<td>150</td>
<td>1.95</td>
<td>13.23</td>
</tr>
<tr>
<td>Sand Lime Brick</td>
<td>SLB</td>
<td>0.22</td>
<td>2.04</td>
<td>0.37</td>
<td>171</td>
<td>1.75</td>
<td>2.3</td>
</tr>
<tr>
<td>Non-Bearing Materials</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Wood</td>
<td>WOD</td>
<td>0.4</td>
<td>1.33</td>
<td>0.1</td>
<td>50</td>
<td>1.2</td>
<td>0.15</td>
</tr>
<tr>
<td>Gypsum Board</td>
<td>GBD</td>
<td>0.26</td>
<td>2.78</td>
<td>0.06</td>
<td>40</td>
<td>0.8</td>
<td>0.013</td>
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<tr>
<td>Sand Cement Plaster</td>
<td>SCP</td>
<td>0.2</td>
<td>3</td>
<td>0.06</td>
<td>86</td>
<td>0.54</td>
<td>0.02</td>
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<td>Mineral Wool</td>
<td>MLW</td>
<td>0.24</td>
<td>0.205</td>
<td>0.17</td>
<td>96</td>
<td>1</td>
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<tr>
<td>Marble</td>
<td>MRB</td>
<td>0.21</td>
<td>8.3</td>
<td>0.1</td>
<td>160</td>
<td>9</td>
<td>0.04</td>
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</table>
The next step is to run the optimization models for each number of layers considering cost only, then considering heat transfer only, then considering both objectives. The selected material/materials based on the dual objectives run is used in the transient heat transfer model to calculate the rate of heat flow through the wall based on the $T_{out}$ (Sol-air temperature) of the surrounding. The $T_{out}$ is calculated using equation (15) according to the location and orientation of the wall, considering the ambient temperature of the typical day of the year at this location. The hourly $T_{out}$ of this case is presented in Table 5. The convection heat transfer coefficients ($h_i$, $h_o$) based on the average wind speed at the chosen location and wall geometry are 1.45 and $3 \frac{Btu}{hr ft^2 \circ F}$, respectively.

<table>
<thead>
<tr>
<th>Time (hr)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{out}$ ($^\circ F$)</td>
<td>72</td>
<td>70</td>
<td>68</td>
<td>66</td>
<td>65</td>
<td>67</td>
<td>73</td>
<td>81</td>
<td>94</td>
<td>106</td>
<td>116</td>
<td>119</td>
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<tr>
<td>Time (hr)</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
</tr>
<tr>
<td>$T_{out}$ ($^\circ F$)</td>
<td>121</td>
<td>112</td>
<td>105</td>
<td>92</td>
<td>85</td>
<td>81</td>
<td>80</td>
<td>78</td>
<td>76</td>
<td>75</td>
<td>74</td>
<td>73</td>
</tr>
</tbody>
</table>

The optimization results show that the designer has four different alternatives to build the wall. The four alternatives resulted from the optimization models and the transient heat transfer model are summarized in Table 6. The table shows that for one layer, the cost optimization model selects the SCB1 with cost 1.1 $/m^2$ and thickness 4 in. While the heat optimization model selects the CCN2 with HBF 13.23 and thickness 5 in. The dual objectives model selected the SCB2 with cost 1.55 $/m^2$ and HBF 9.65 and thickness 8in. The properties of this material are then applied to the transient heat transfer model to give $q''$ 126.4 Btu/ft².

For two layers, the cost optimization model selects the SCB1+SCP with cost 1.64 $/m^2$ and thickness 4.8 in. While the heat optimization model selects the CCN2+CCN2, which means that the two layers are of the same material, with HBF 26.46 and thickness 10 in. The dual objectives model selected the CCN2+SCP with cost 2.49 $/m^2$ and HBF 13.25 and thickness 10in. The properties of these materials are then applied to the transient heat transfer model to give $q''$ 99.9 Btu/ft².

For three layers, the cost optimization model selects the SCP+SCB1+SCP with cost 2.18 $/m^2$ and thickness 5.6 in. While the heat optimization model selects the CCN2+MWL+CCN2, with HBF 30.52 and thickness 12 in. The dual objectives model selected the SCP+SCB2+MWL
with cost 3.09 $/m^2 and HBF 13.73 and thickness 10.8 in. The properties of this material are then applied to the transient heat transfer model to give \( q'' = 67.1 \text{ Btu/ft}^2 \).

For four layers, the cost optimization model selects the SCP+SCB1+SCP+SCB1 with cost 3.28 $/m^2 and thickness 9.6 in. while the heat optimization model selects the WOD+SCB1+CCN2+SCP, with HBF 26.63 and thickness 12 in. The dual objectives model selected the CCN2+SCP+SCB1+MWL with cost 4.59 $/m^2 and HBF 18.34 and thickness 11.8 in. The properties of this material are then applied to the transient heat transfer model to give \( q'' = 41.6 \text{ Btu/ft}^2 \). Afterwards, more layers can be tested by the same way.

The transient heat transfer model predicts the temperature of the inner layer of the wall throughout the whole day, which can be used to draw a graph to show how the temperature varies with time for the four alternatives, as shown in Figure 12.

<table>
<thead>
<tr>
<th>Number of Layers</th>
<th>Cost optimization model</th>
<th>Heat optimization model</th>
<th>Dual objectives model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( Z_c ) $/m^2</td>
<td>\Delta x ) in</td>
<td>( Z_h ) $/m^2</td>
</tr>
<tr>
<td>1</td>
<td>SCB1</td>
<td>1.1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>SCB1</td>
<td>1.64</td>
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<td>SCP</td>
<td>3.28</td>
<td>9.6</td>
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<tr>
<td>5</td>
<td>SCP</td>
<td>3.82</td>
<td>10.4</td>
</tr>
</tbody>
</table>

Table 6: Summary of optimization results
The conditioning costs over a certain period of time (for example 50 years) can be compared to the initial cost of the wall for area of 1 \( m^2 \). The present worth value of this annual conditioning payments is calculated based on average inflation rate 8\% [27] using equation (24). The results of the present worth value for air conditioning and material costs are summarized in Table 7.

\[
A = P \times \frac{i(1+i)^n}{(1+i)^n-1} \quad [28]
\]

**Table 7 Present worth value for air conditioning and material costs [29]**

<table>
<thead>
<tr>
<th>Number of layers</th>
<th>Q Through the wall/day (kWh/ft(^2))</th>
<th>Material cost (Present Value)</th>
<th>Present value of annual cooling over 50 years</th>
<th>Total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.037</td>
<td>1.55</td>
<td>13.22</td>
<td>14.774</td>
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<td>0.0293</td>
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<td>0.0197</td>
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<td>7.04</td>
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<td>4</td>
<td>0.0122</td>
<td>4.59</td>
<td>4.36</td>
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<td>5</td>
<td>0.0121</td>
<td>5.2</td>
<td>4.32</td>
<td>9.5246</td>
</tr>
</tbody>
</table>
Figure 13 shows the present worth value for air conditioning and material costs for different layers numbers. The figure shows that the four layers wall is the best choice regarding both the heat transfer and initial cost of the building.

Figure 13: Present worth value for air conditioning and material costs
Chapter 5 Conclusion and Recommendations:

5.1. Conclusion

The main goal of this research is to develop an approach to select the materials for building a composite building wall considering two objectives, which are minimum cost and minimum heat transfer through the wall. These two objectives are of high necessity to the designers as the initial cost of the materials affects the whole initial investment cost directly, and the heat transfer into or out of the wall also affects the heating and cooling loads required in the room, which affects the running costs of the building.

Designers have numerous materials with different economic, physical and thermal properties to choose from, so the first step in this approach is to list all the available materials in the market with their properties. Then get the ambient temperature of the typical day of the year according to the location of the building and calculating the Sol-Air temperature (Equation 25) of the typical day considering the orientation of the wall.

The composite building wall can be constructed of one, two, three, four layers or more. The model studies each number of layers separately, then the optimum combination of materials to be used for each number of layers is presented to the designer with the relative cost and heat transfer through the wall to select which alternative to utilize for the building. The Network for each number of layers is drawn considering the bearing factor, that there should not be two consecutive non-bearing layers. Then Pareto optimization is applied, which starts by constructing a model that considers the cost objective only and selecting the materials that would be the most economic, then constructing another model considering the heat transfer though the walls only, finding out the minimum amount of heat that can pass through the wall ignoring cost. Finally, the dual objectives model is constructed considering both factors (cost and heat transfer), which searches for the materials that can give the minimum deviation from the optimum cost and optimum heat transfer. After finding the optimum materials to be used for each number of layers, the transient heat transfer through the wall is calculated based of the Sol-Air temperature of the typical day of the year at the location where the wall is to be constructed. The transient heat transfer model applies Crank-Nicholson technique, which is a finite difference method that is constructed on MATLAB [24] for the different number of layers. The Data are presented finally to the designer.
showing the optimum materials to be used for each number of layers, with the relative cost and transient heat transfer through this combination of materials.

The results of this approach also consider the thickness limitations of the wall stated by the designer according to the space limitations or the functionality of the building being residential or non-residential. The dual objective optimization model is based on equivalent necessity of both objectives (cost and heat transfer), although dominance of one factor over the other can be applied to the model. The heat transfer model is verified using the Transient heat transfer model for multiple layers wall in ASHRAE [5] “Equation (26)”. The approach followed in this research is implemented in the previous section considering actual building materials in the market with their corresponding properties, and the results are presented in table 2. This approach can also be used for more number of layers in case needed, not just for construction walls, but also for other purposes like walls of furnace chambers.

5.2. Recommendations for future work

The following recommendations are listed for future work to extend the presented research work:

1. The designer can give the actual dimensions of the whole building such that the model can calculate the actual heat transfer through it, rather than the heat transfer through 1m² of area.
2. The transient heat transfer model can be integrated to worldwide temperature map, that the designer can just give the location of the building without calculating the Sol-Air temperature as an input to the model.
3. Other objectives can be achieved at the same time in addition to the cost and the thermal efficiency, by introducing more variables into the Pareto optimization model.
4. Other constraints can be added to the model like chemical stability, water resistance or the ability to withstand external environmental conditions.
5. Other types of composites can be used rather than laminates composites, like fiber composites, flake composites or particulates filled composites.
References:


function[]=thermocalc3(c1, k1, row1, c2, k2, row2, c3, k3, row3, Tin, Tout, L1, L2, L3, hi, ho, dt,TS)

Qt=0;
x=0;
R1=(1/hi);
R2=(0.5*L1/k1);
Ra=R1+R2;
R3=R2;
R4=(0.5*L2/k2);
Rb=R3+R4;
R5=R4;
R6=(0.5*L3/k3);
Rc=R5+R6;
R7=R6;
R8=(1/ho);
Rd= R7+R8;
CM1=(row1*L1*c1/dt);
CM2=(row2*L2*c2/dt);
CM3=(row3*L3*c3/dt);
T1=TS;
T2=TS;
T3=TS;

for j=1:24
  for i=1:40
    T1p=(((Tin-T1)/(CM1*Ra))+((T2-T1)/(CM1*Rb)))+T1;
    T2p=(((T1-T2)/(CM2*Rb))+((T3-T2)/(CM2*Rc)))+T2;
    T3p=(((T2-T3)/(CM3*Rc))+((Tout(j)-T3)/(CM3*Rd)))+T3;
    Q=dt*0.5*(((T1-Tin)/Ra)+(T1p-Tin)/Ra);
    T1=T1p;
    T2=T2p;
    T3=T3p;
    Qt=(Qt+Q);
  end
  disp(T1)
  x=x+1;
end

i=1;
end

disp(Qt);
disp(T1);
end
Appendix B: Program Code for four walls

function[]=thermocalc4(c1, k1, row1, c2, k2, row2, c3, k3, row3, c4, k4, row4, Tin, Tout, L1, L2, L3, L4, hi, ho, dt,TS)

    Qt=0;
    x=0;
    R1=(1/hi);
    R2=(0.5*L1/k1);
    Ra=R1+R2;
    R3=R2;
    R4=(0.5*L2/k2);
    Rb=R3+R4;
    R5=R4;
    R6=(0.5*L3/k3);
    Rc=R5+R6;
    R7=R6;
    R8=(0.5*L4/k4);
    Rd=R7+R8;
    R9=R8;
    R10= (1/ho);
    Re= R9+R10;
    CM1=(row1*L1*c1/dt);
    CM2=(row2*L2*c2/dt);
    CM3=(row3*L3*c3/dt);
    CM4=(row4*L4*c4/dt);
    T1=TS;
    T2=TS;
    T3=TS;
    T4=TS;

    for j=1:24
        for i=1:40
            T1p=(((Tin-T1)/(CM1*Ra))+((T2-T1)/(CM1*Rb)))+T1;
            T2p=(((T1-T2)/(CM2*Rb))+((T3-T2)/(CM2*Rc)))+T2;
            T3p=(((T2-T3)/(CM3*Rc))+((T4-T3)/(CM3*Rd)))+T3;
            T4p=(((T3-T4)/(CM4*Rd))+((Tout(j)-T4)/(CM4*Re)))+T4;
            Q=dt*0.5*(((T1p-Tin)/Ra)+(T1p-Tin)/Ra);
            T1=T1p;
            T2=T2p;
            T3=T3p;
            T4=T4p;
        end
    end

    Qt=(Qt+Q);
end
disp(T1)

x=x+1;

i=1;
end

disp(Qt);
disp(T1);
end